Modeling USDEUR FX Swaptions: A Comparative Analysis

Black '76 vs. Merton Jump-Diffusion Models

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Pricing a Cross-Currency Derivative

The project focuses on valuing a USDEUR FX Swaption—the right to enter an FX swap at a fixed future rate (K).

Objective: Quantify the impact of Jump Risk by comparing the industry standard against a fat-tailed model.

Feature	Black '76 (Benchmark)	Merton Jump-Diffusion
Volatility	Constant (σ)	Stochastic, Jumps (σ, λ, μ_J)
Distribution	Log-Normal (Thin Tails)	Leptokurtic (Fat Tails)
Pricing Method	Closed-Form Formula	Monte Carlo Simulation
Result	Underprices Tail Risk	Captures Jump Risk Premium

Table: Comparison of Volatility Models

Valuation Models: Core Formulas

- Black '76 (Benchmark Model): Closed-form solution assuming log-normal diffusion for the Forward Rate (F).
  - Pricing Formula: $V_{\text{Black}} = P_{\text{fixed}} \cdot [FN(d_1) - KN(d_2)]$

$$d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T_{\text{exp}}}$$

- Merton Jump-Diffusion (Alternative Model): Prices derived via Monte Carlo Simulation of the asset's Stochastic Differential Equation(SDE).
  - Stochastic Process (SDE):

$$\frac{dS_t}{S_t} = (\mu_{\mathbb{Q}} - \lambda j)dt + \sigma_{\text{JD}} dW_t + dJ_t$$

  - Pricing (Monte Carlo Estimate):

$$V_{\text{JD}} = D(T)P_{\text{fixed}} \cdot \mathbb{E}^{\mathbb{Q}}[\max(S_T - K, 0)]$$

5-Step Valuation Pipeline

1. **Data Collection**: Source FX Spot Rate (S_0), USD Par Yields (FRED), and EUR Market Rates.
2. **Curve & Smile Construction**:
    1. Bootstrapped USD/EUR Par Yields to Continuous Zero Curves.
    2. Interpolated FX Option quotes (ATM, Risk Reversal, Butterfly) using Vanna-Volga to build the volatility surface.
3. **Valuation & Comparison**:
    1. Black '76 (Closed-Form Solution) $\rightarrow$ Benchmark Price.
    2. Merton JD (Monte Carlo simulation) $\rightarrow$ Risk-Adjusted Price.

Comparative Pricing Results

Market Inputs:

- Spot Rate (S_0): 1.1519
- Forward Rate (F): 1.1497
- Strike Rate (K): 1.0500 (*Out-of-the-Money*)
- Option Expiry (T): 2.0 Years

Model	**Calculated Price (USD)**
Black '76 (Benchmark)	0.094153
Merton Jump-Diffusion (MC)	0.101742

Table: Calculated Swaption Price (USD Domestic Currency)

Price Difference (JD − Black)

+\$0.007589 (an increase of 8.06% over Black)

Inference: Why the Merton JD Model is Superior

Finding 1: Quantifying the Jump Risk Premium

- The higher JD price (+$**0.0076** premium) explicitly values the risk of sudden, large movements in the underlying FX rate.
- The JD price is a **more prudent, risk-adjusted valuation** that reflects true market volatility expectations.

Finding 2: Theoretical Consistency

JD explains the Volatility Smile and Tail Risk.

- The Merton model's fat-tailed distribution naturally assigns a higher probability to extreme OTM payoffs.
- **Conclusion:** The Black '76 model systematically underprices this swaption because it fundamentally ignores the tail risk inherent in FX markets.

Thank You!